What does the Non-Mathematics Intensive Major Think *Mathematics* Is?

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Abstract

This study explores the myriad definitions and conceptualizations of mathematics, with a particular focus on how undergraduate students in a mathematics course designed for non-mathematics intensive majors define the term. Students enrolled in this particular course at a regional university in the Midwest were asked the open-ended question: "What is math?" The researchers employed content analysis in their analysis and interpretation of the data. A number of categories emerged, with the modal category being the conception of mathematics as problem solving that occurs in mathematics classes.

Keywords: mathematics, problem-solving, definitions

What does the Non-mathematics Intensive Major Think *Mathematics* Is?

Why is mathematics important? Should all secondary students have to take mathematics? What kind of technology should be used in the teaching of mathematics? What are the goals of mathematics education? What are best practices in the teaching of mathematics? How should students be taught to study mathematics? Is the mathematics learned in academic courses applicable to life outside of the classroom?

While each of these questions (and more) are important questions in the field of mathematics education, we suggest that the answer to each of them is dependent on what is *meant* by mathematics. What one thinks mathematics is informs and shapes one's answers. If one thinks mathematics is a collection of arithmetic facts studied in school, then one might view appropriate mathematical study as the memorization of those facts and may not turn to mathematics to help solve situations in one's own life.

There are numerous answers to the question, what is mathematics? Some people define mathematics as a *study*, and then include various aspects that they understand to be a part of some mathematics classes. For example, mathematics is the study of numbers. Mathematics is the study of shape. Mathematics is the study of motion. The definition of mathematics under this view is constantly changing as discoveries are made. Mathematics, then, might be the collection of topics that mathematicians produce.

Others define mathematics as a *tool*. Some believe that everyday people use mathematics in everyday life. Perhaps this view defines mathematics as mainly arithmetic, or perhaps it includes areas of statistics and data analysis which might also be used in everyday life. Describing mathematics as a tool may also mean a tool that engineers and scientists use.

Others go so far as to say that the whole world is mathematical, and thus mathematics is *an explanation of how the world works*. In this more philosophical view, mathematics is a study that develops critical, logical, and/or quantitative thinking. Mathematics might be viewed as a study that instills the power of abstract thought into its students. Mathematics uses symbols and abstraction to generalize from arithmetic, and so some people define mathematics as a special language. Others emphasize the structural nature of mathematics, and define mathematics to be the study of structure, while still others think of mathematics as a work of art, or a formal game with very precise rules.

Literature Review

Even among mathematicians, there are differing views on what mathematics is, and there is no one right answer. Elementary children believe that mathematics is arithmetic, counting, and/or the four operations of adding, subtracting, multiplying, and dividing (Kouba & McDonald, 1991). Pre-service elementary teachers view mathematics as conducting calculations (Latterell, 2012). Mathematics majors who are going to be secondary teachers view mathematics as much more than just calculations in that it includes following rules, problem solving, searching for patterns, and thinking (Latterell & Wilson, 2002). Current teachers view mathematics as numbers (Duatepte Paksu, 2008).

The questionnaire included in *Conceptions of Mathematics* (Crawford, Gordon, Nicholas, Prosser, 1994, 1998a, 1998b) was designed to measure what university mathematics students think mathematics is. The conclusion from their studies was that students thought mathematics was one of five things:

1. Math is numbers, rules, and formulas.
2. Math is numbers, rules, and formulas, which can be applied to solve problems.
3. Math is a complex logical system; a way of thinking.
4. Math is a complex logical system, which can be used to solve complex problems.
5. Math is a complex logical system, which can be used to solve complex problems, and provides new insights used for understanding the world.

The first two responses were labeled by the researchers as fragmented conceptions and the last three as cohesive. In addition, the researchers classified students as having a surface approach or a deep approach to the learning of mathematics, and found that 91% of those with a fragmented conception of math had a surface approach to learning math, while 90% of those with a cohesive conception of math had a deep approach to learning math.

Lim Chap Sam (1999) surveyed 548 adults as to what they think mathematics is. The answers emerged in five categories:

1. An answer concerning an attitude towards mathematics (e.g., Mathematics is boring.).
2. An answer concerning mathematics ability (e.g., Mathematics is hard.).
3. An answer concerning the process of learning mathematics (e.g., Mathematics is problem solving.).

4. An answer concerning the nature of mathematics (e.g., Mathematics is numbers and equations.).
5. An answer concerning the values and goals in mathematics education (e.g., Mathematics is beautiful, fun, a mystery, and/or a challenge.).

Fida Atallah (2003) surveyed 238 female students at a university in the Middle East and found that 37% viewed mathematics as a school subject used in everyday life and at work; 18% viewed mathematics as a mental exercise to develop intellectual ability; 10% viewed mathematics as numbers and rules for doing calculations; 9% viewed mathematics as a school subject used in learning other subjects; 5% viewed mathematics as a symbolic language; 4% viewed mathematics as a form of art; and finally 1% viewed mathematics as a language of science. Over 80% of the responses viewed arithmetic as the most useful subject within mathematics.

A group of international colleagues (Wood, et al., 2011) found that university students' perceptions of mathematics was hierarchical, with students viewing mathematics as "an approach to life and a way of thinking," or "about building and using models," or as "a toolbox of individual components and procedures, perhaps only numerical calculations" (p. 101). In an extension of their study, they also interviewed students to see what students thought their future use of mathematics would be. Many students simply had no idea, and others mentioned some type of procedural skills (a view of mathematics as a toolbox), and others mentioned conceptual skills (a view of mathematics as a way of thinking). A few students mentioned mathematics as playing a major role in their career (e.g., someone who wanted to be a statistical consultant answered this way).

Method

This study asks how students from academic majors that do not require mathematics courses view mathematics. At this regional university in the Midwest, many majors require at least one course in mathematics. For those majors that do not, the university requires a course in a category that includes mathematics, logic, and critical thinking. If a student whose major does not require mathematics wants to take a course such as Calculus I, he/she may do so. But, many of these students do not have a very strong background in mathematics, and prefer a course that does not require algebra. The institution offers a course entitled *Contemporary Mathematics* that looks at uses of mathematics in the world, but requires very little previous mathematics ability (basically only pre-algebra ability). The students enrolled

in this course might be majoring in art, sociology, or history, among other majors in the liberal arts or humanities.

The researchers asked the 58 students enrolled in *Contemporary Mathematics* on the first day of class to write an answer to the question, "What is math?" It is not part of this study to attempt to influence their answers, and thus, the researchers presented the question on the very first day. The two researchers separately categorized the responses. Each researcher placed together those responses that seemed similar and created titles for the resulting categories. The researchers then met and compared the codes and the placement of answers into those codes.

Before discussion, the researchers were in 83% agreement. That is, on 83% of the responses, the placement of the responses with other responses matched. At this point, the researchers agreed to category names (which was usually a combination of the names that the researchers were separately using) and the researchers discussed one by one the responses for which there was disagreement. Eventually, there was 100% agreement with the following categories and placement of responses in them:

- Mathematics is a vehicle for discovering and explaining the world.
- Mathematics is a subject that deals with numbers.
- Mathematics is problem solving.

One response was left uncategorized. These categories will be discussed in detail in the results section.

The problem-solving category originally caused the most disagreement. It was decided to attempt to separately re-categorize the responses in this category into three sub-categories:

- Mathematics is problem solving that is needed on a day-to-day basis to make life work.
- Mathematics is problem solving that is used by professionals, such as engineers.
- Mathematics is problem solving that occurs in mathematics classes.

In essence, the researchers believed that students were using the term "problem solving" in fundamentally different ways. Some students used it to refer to the problem solving that occurs in daily life. Others meant that only certain professionals do problem solving, while still others thought that problem solving occurred only in mathematics classes (and had no real-life applications). The researchers attempted to go back through the problem solving answers and separate them into these three

categories, but unfortunately, the researchers found this difficult to do, as student responses were often difficult to interpret.

In an attempt to better understand what students thought problem solving was, the researchers surveyed the students a second time. This second survey occurred approximately two weeks into the class. The researchers asked each of the students to select one of two responses given below, whichever one they thought was the most true.

1. The purpose of mathematics is to learn real-life skills. When we solve problems, we solve them so that when we later encounter these problems in real-life, we can solve them then. School mathematics is for problems that most people encounter in life. More advanced mathematics solves more advanced real-life problems, such as occurring in engineering or science.
2. The purpose of mathematics is to solve problems whether they have an application or not. When we solve problems, we solve them for the sake of learning to solve them. Some of the problems do not even have a real-life application.

Three-fourths (75%) of the students selected the first choice. Based on this, the researchers decided to go back through the problem solving and separate it into just two categories:

- Mathematics is problem solving that is needed on a day-to-day basis to make life work, including being used by professionals, such as engineers, to solve problems.
- Mathematics is problem solving that occurs in mathematics classes.

The researchers had a 77% agreement rate the first time through, and a 100% agreement rate after discussion. Thus, our coding resulted in four categories. In Table 1, the categories are given, with the percent and number of responses in each category.

Table 1: Categories with Percent and Number of Responses

Category	Percent of Responses	No. of Responses
Mathematics is a vehicle for discovering and explaining the world.	8.6%	5
Mathematics is a subject that deals with numbers.	34.4%	20

Mathematics is problem solving that is needed on a day-to-day basis to make life work, including being used by professionals, such as engineers, to solve problems.	17%	10
Mathematics is problem solving that occurs in mathematics classes.	38%	22

Results

Only one response was left outside of a category, and it stated, “mathematics is intended to help issues/or problems of money.” This response might have been placed in the everyday life problem solving category, but it was so specifically about money that the researchers left it in a category all on its own.

Five students (8.6%) gave responses that the researchers categorized as viewing mathematics as a vehicle for discovering and explaining the world. Mathematics solves the “world’s mysteries,” and answers “bigger questions.” A representative response states, “Math is humanity’s way to explain the world. Mathematicians solve the world’s mysteries.”

Twenty students (34.4%) believe that mathematics is a subject that deals with numbers. Two typical responses follow. “To me math is all about gathering a group of numbers to get more numbers, either a larger or smaller number.” “When I think of math, I think of it as the study of numbers. Adding, subtracting, multiplying numbers. Math is numbers.”

The final two categories viewed math as solving problems. Ten students (17%) viewed mathematics as problem solving that is used on a day-to-day basis by regular people and certain professionals (e.g., engineers). For example, one student stated, “Math is a process of using numbers to solve problems or create something that is useful and not just numbers without a meaning.” Another wrote, “Math is something most people use on a daily basis, usually it is very simple. Math is used to solve problems, whether small or large.”

The remaining 22 students (38%) viewed mathematics as a class in which one worked on problems, attempting to solve them, but these problems seem to have no application to real-life. The purpose for solving the problems is a bit of a mystery, as few of the responses connected this solving to some purpose, such as developing reasoning skills. Those who did discuss a purpose or reason seemed to claim that the purpose was simply to do it. For example, one student wrote, “The purpose is to come up

with a result." Another student wrote, "For the reason of finding a solution of some kind." The following response comes close to viewing mathematics as a game, "Math is using strategic methods to solve numerical problems." The lack of purpose and circular nature of this category can be seen in this response: "Math is solving problems that in order to solve these problems one needs to use math techniques."

In sum, most students view mathematics as the solving of mathematics problems (n = 22), and close behind is the view that mathematics is a school subject that studies numbers (n = 20). Fewer students view mathematics as having everyday value in problem solving (n = 10), or even explaining the entire world (n = 5). The researchers left the one response about math intending to help with issues of money as unclassified. Thus, 42 of the students, which is 72% of the students, view mathematics as a classroom subject.

Limitations, Implications, and Further Study

Two possible limitations arose in this study. The second survey had a slight time lag, and it is possible that course instruction influenced responses. However, the second survey was only used to determine whether problem solving should be split into two or three categories and should not have had significant effect on our ability to paint a picture of mathematics beliefs.

The more significant limitation is the obvious one that the sample was one class at one point in time, and study participants were not randomly selected. It is certainly possible that these students are unique in a variety of manners. While a sample size of 58 respondents is not overly small, it nevertheless remains a snapshot in time.

Assuming repeated studies would give similar results, the natural question is where this study could lead. The main point of the study was to describe what majors in non-mathematics-intensive fields think mathematics is. Although this is philosophically interesting in and of itself, are there possible practical implications of knowing this? The researchers suggest that knowing what these majors think mathematics is can have implications on all of the following questions:

- What are the appropriate collegiate mathematics requirements, if any, for these majors?
- If a requirement were made, what content or processes would this course contain?
- Is it possible to change the definition of mathematics for these majors?

- If it is possible to change the definition of mathematics, is it a desirable goal?

It is beyond the scope of this study to answer these questions, and thus, our list of implications is also our list for further research possibilities. In addition, the researchers do not suggest that research is lacking on these questions (see Hastings, 2006), only that knowing these views of mathematics can have an influence on this research. For example, when faculty at undergraduate institutions are deciding on general education requirements, if they decide to require some sort of mathematics, what do they really mean by mathematics? Knowing that different constituents mean different things and in particular what majors in non-intensive math fields think mathematics is could have influence on what should be required and how it should be packaged. This has direct relevance for those institutions that may be re-vamping their general education requirements.

Further developing our example, some institutions require a course in quantitative reasoning, and often these are offered from the mathematics faculty. However, many mathematicians (see the work of Lynn Arthur Steen) argue that quantitative reasoning courses are not courses in mathematics. This may be fine. But, if these non-intensive math majors take such a course, how would it fit with their definitions of math? A pre- post test study may determine if their definitions change. The researchers contend that their definitions of mathematics may move in the direction of the discipline being an everyday tool.

Although there is not a single “correct” definition of mathematics, and perhaps no dire need to try changing the ways people define it, the researchers contend that it is important and worthwhile to understand just what *is* meant when people say “math.” The researchers find that many people, including those in our study, often define the discipline rather narrowly. General education mathematics courses could serve to broaden students’ views of the subject of mathematics, thereby introducing students to a more multifaceted and fascinating discipline than they had previously imagined. To the extent that educators could accomplish this in our undergraduate general education courses, they would be inviting people into the discipline of mathematics – not necessarily convincing everyone to become math majors, but demonstrating the myriad uses, applications, and features of the field and thereby fostering an enhanced understanding and appreciation of mathematics.

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